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approaches infinity the larger circle approaches a straight line and the construction merges into the classic construction for the cissoid.

II. Relating to a Geometric Proof of a Theorem on Collineations.

By James H. Weaver, Hilliard, Ohio.

The relations between double elements of a collineation in space (by which is meant a three-space, unless otherwise stated) have been discussed analytically, use being made of the characteristic equation of the collineation and of the properties of elementary divisors.[1] Enriques, in his *Projective Geometrie* (Leipzig, 1903), has shown geometrically that if a collineation in a plane has a double point it has a double line. It is here proposed to prove the

Theorem. *If a collineation in space has a double point, it has a double plane.*

If the collineation is perspective the theorem is evident. Suppose, then, that the collineation is non-perspective.

Let A be a double point of a non-perspective collineation π and let a be a line through A. Then π sets up the following correspondence

$$a \barwedge a_1 \barwedge a_2 \barwedge a_3 \barwedge a_4.$$

Let B and C be two distinct points on a different from A. Then

$$B \barwedge B_1 \barwedge B_2 \barwedge B_3 \barwedge B_4,$$

where B_i is on a_i ($i = 1, 2, 3, 4$), and similarly for the C's. Let the intersection $(BB_1, CC_1) = D$. Since A is a self-corresponding point, D will be independent of the points B and C. Also, let $(B_iB_j, C_iC_j) = D_i$, ($i = 1, 2, 3, j = i + 1$).

If a, a_1 and a_2 are coplanar the theorem is evident and the double plane contains also a double line.

If a, a_1 and a_2 are not coplanar, D_1 is not on plane $[aa_1]$. Let the line DD_1 meet the plane $[aa_1]$ in some point E, and let us assume C so chosen that E lies on CC_1. Let $\pi(E) = E_1$. E_1 is on D_2D_3 and also on the plane $[CC_1, C_1C_2]$. Hence D, D_1, D_2 and D_3 are coplanar and the plane containing them is a double plane.

By duality, *if a collineation in space has a double plane it also has a double point.*

The method here used for establishing the theorem for a three-space is general and can be applied to an n-space giving the theorem that if a collineation in an n-space has a double point it has a double $(n - 1)$-space. This method of proof when applied to collineations in a plane is somewhat simpler than that of Enriques mentioned above.

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[1] Segre, "Sulla teoria e sulla classificazione delle omografie in un spazio lineare ad un numero qualunque di dimensioni," *Reale Accad. dei Lincei*, Serie 3*a*, Bd. XIX, S. 6. See also Muth, *Theorie und Anwendung der Elementartheiler*, Leipzig, 1899; Veblen and Young, *Projective Geometry*, Vol. I, Boston, 1910; Bôcher, *Introduction to Higher Algebra*, N. Y., 1912; Newson, "A New Theory of Collineations," *Amer. Jour. Math.*, Vol. XXIV.